(3) A simple sample of heights 6,400 Englishmen has a mean of 170 inches and a SD of 6.4 inches, while a simple sample of heights of 1600 Australians has a mean of 172 inches and a SD of 6.3 inches. Do the data indicate that Australians are on the average taller than Englishmen?

Soln:

$n_1 = 6400$ $\quad \bar{x}_1 = 170$, $\quad S_1 = 6.4$

$n_2 = 1600$ $\quad \bar{x}_2 = 172$, $\quad S_2 = 6.3$

$\mu_1 \rightarrow$ mean height of the population of Englishmen

$\mu_2 \rightarrow$ mean height of the population of Australians.

1. $H_0 : \mu_1 = \mu_2$

2. $H_1 : \mu_1 < \mu_2$

3. $\alpha = 1\%$ and $\alpha = 5\%$.

4. Z_α at 1% 2.33 & Z_α at $5\% = 1.645$

5. Test Statistic $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$

$$z = \frac{170 - 172}{\sqrt{\dfrac{(6.4)^2}{6400} + \dfrac{(6.3)^2}{1600}}} = -11.32$$

Conclusion

$|z| = 11.32$

At 1% LOS, Cal z $\nless$ Tab z Reject Ho

At 5% LOS, Cal z $\nless$ Tab z Reject Ho.

Hence accept H_1

We conclude that Austrialians on the average taller than Englishmen.

4 A sample of 100 bulbs of brand A gave a mean lifetime of 1200 hrs with a SD of 70 hrs, while another sample of 120 bulbs of brand B gave a mean lifetime of 1150 hrs with a SD of 85 hrs. Can we conclude that brand A bulbs are superior to brand B bulbs?

Soln:

$n_1 = 100 \quad \bar{x}_1 = 1200 \quad S_1 = 70$

$n_2 = 120 \quad \bar{x}_2 = 1150 \quad S_2 = 85$

1. $H_0 : \mu_1 = \mu_2$
2. $H_1 : \mu_1 \neq \mu_2$
3. LOS $1\% = \alpha$
4. $Z_\alpha = 2.33$ at 1%
5. Test statistic $z = \dfrac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$

$$z = \frac{1200 - 1150}{\sqrt{\dfrac{70^2}{100} + \dfrac{85^2}{120}}} = \frac{50}{\sqrt{49 + 60.208}}$$

$$z = 4.79$$

6. Conclusion:

$|z| \not< Z_\alpha \quad$ cal $z \not<$ Table z

We reject H_0

Hence accept H_1.

Hence brand A bulbs are superior to brand B.

5. Two samples drawn from different populations gave the following results:

	Size	mean	SD
Sample I	100	582	24
Sample II	100	540	28

Test the hypothesis, at 5% LOS, that the difference of the means of the population is 35.

Soln:

$n_1 = 100$ $\quad \bar{x}_1 = 582,$ $\quad S_1 = 24$.

$n_2 = 100,$ $\quad \bar{x}_2 = 540,$ $\quad S_2 = 28$

$\mu_1 - \mu_2 = 35$

1. $H_0 : \mu_1 - \mu_2 = 35$

2. $H_1 : \mu_1 - \mu_2 \neq 35$ (Two tailed test)

3. $\alpha = 5\%$

4. $Z_\alpha = 1.96$

5. Test Statistic $z = \dfrac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

$$z = \frac{(582 - 540)}{\sqrt{\dfrac{24^2}{100} + \dfrac{28^2}{100}}} = \frac{7}{3.7} = 1.90$$

6. Conclusion :- cal $z <$ Tab z.

We accept H_0.

6. The mean height of 50 male students who showed above average participation in college athletics was 68.2 inches with a SD of 2.5 inches, while 50 male students who showed no interest in such participation had a mean height of 67.5 inches with a SD of 2.8 inches. Test the hypothesis that male students who participate in college athletics are taller than other male students.

Soln.

$n_1 = 50$ $\quad \bar{x}_1 = 68.2$ $\quad s_1 = 2.5$

$n_2 = 50$ $\quad \bar{x}_2 = 67.5$ $\quad s_2 = 2.8$

$\mu_1 \rightarrow$ mean of the male students participate in college athletics

$\mu_2 \rightarrow$ mean of the other male students.

1. $H_0: \mu_1 = \mu_2$
2. $H_1: \mu_1 > \mu_2$ (Right tailed test)
3. LOS = 1% or 5%
4. $z_\alpha = 2.33$ $\quad z_\alpha = 1.645$
5. Test Statistic $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

$$z = \frac{(68.2 - 67.5)}{\sqrt{\frac{(2.5)^2}{50} + \frac{(2.8)^2}{50}}} = \frac{0.7}{0.5} = 1.32$$

6. Conclusion :

Cal z < Table z at 1% & 5%

We accept H_0. It means we reject H_1

Hence Athletic male students are not taller than other male students.

Testing of Hypothesis for Small Sample ($n < 30$)

t - test i.e (or) Student's t - test

Let $x_1, x_2, \ldots x_n$ be a random sample of size n from a normal population with mean μ and variance σ^2.

The Student's t test is defined in the statistics as

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}, \quad \text{where}$$

$$\bar{x} = \frac{\Sigma x_i}{n} \rightarrow \text{sample mean}$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \rightarrow \text{Pop variance } \sigma^2$$

and $t \sim$ Student's t-distribution with $\nu = n-1$ degrees of freedom.

Properties:

* The range of t-distribution is from $-\infty$ to ∞.
* t-distribution is symmetrical about $t = 0$ and has a mean $= 0$
* Variance $= \dfrac{\nu}{\nu - 2}$ if $\nu > 2$ and $\mu_2 > 1$ always.

Applications of t-distribution:

* It is used to test the significance of the difference of sample mean and the population mean.
* It is used to test the significance of difference between two sample mean.

Test for Specified mean :-

Test Statistics

$$t = \frac{(\bar{x} - \mu)}{\frac{s}{\sqrt{n-1}}}, \text{ where } \quad \bar{x} - \text{Sample mean}$$

μ - pop. mean

s = Sample SD

n - Sample size.

Problems:

The mean lifetime of a sample of 25 bulbs is found as 1550 hrs with SD of 120 hrs. The Company manufacturing the bulbs claims that the average life of their bulbs is 1600 hrs. Is the claim acceptable at 5% LOS ?

Soln:

$n = 25 < 30$ (use 't' - test)

$\bar{x} = 1550 \qquad s = 120$

$\mu = 1600$

(1) $H_0 : \mu = 1600$

(2) $H_1 : \mu < 1600$

(3) $\alpha = 5\%$ with $df = n-1 = 24$

(4) Table value of $t = 1.711$

(5) Test Statistic $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$

$$t = \frac{1550 - 1600}{\frac{120}{\sqrt{24}}} = \frac{-50}{24.53}$$

$t = -2.04$

$|t| = 2.04$

(6) Conclusion : Cal $t \not< $ Tab t

So we reject H_0. Hence we accept H_1

The claim of the Company is not acceptable at 5% LOS.

Scanned by CamScanner

2. A sample of 10 house owners is drawn and the following values of their incomes are obtained with mean Rs. 6000 and SD Rs. 650. Test the hypotheses that the average income of house owner of the town is Rs. 5500.

Soln:

$n = 10 < 30$ $\quad \bar{x} = 6000$ $\quad SD = 650 = S$

$\mu = 5500$

1. $H_0 : \mu = 5500$

2. $H_1 : \mu \neq 5500$

3. $\alpha = 5\%$

4. Table Value at 5% with $n-1 = 10-1 = 9$ df is 1.833

5. Test Statistic

$$t = \frac{(\bar{x} - \mu)}{\frac{S}{\sqrt{n-1}}}$$

$$= \frac{(6000 - 5500)}{\frac{650}{\sqrt{9}}} = \frac{500}{\frac{650}{3}}$$

$t = 2.3076$

6. Conclusion cal t > Table t.

We reject H_0.

Hence we accept H_1.

i.e, The average income of the house owner of the town is not equal to 5500.

Scanned by CamScanner

3. A random sample of 10 boys had the IQ's : 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of the population mean IQ of 100?

Soln:

$n = 10 < 30$

$\mu = 100$

$$\bar{x} = \frac{70+120+110+101+88+83+95+98+107+100}{10}$$

$$\bar{x} = 97.2$$

x :	70	120	110	101	88	83	95	98	107	100
$x-\bar{x}$:	-27.2	22.8	12.8	3.8	-9.2	-14.2	-2.2	0.8	9.8	2.8
$(x-\bar{x})^2$:	739.84	519.84	163.84	14.44	84.64	201.64	4.84	0.64	96.04	7.84

$\Sigma(x-\bar{x})^2 = 1833.6$

WKT $\quad s^2 = \dfrac{\Sigma(x-\bar{x})^2}{n-\cancel{1}} = \dfrac{1833.6}{10-\cancel{1}} = \cancel{203.73}\ 183.36$

$s = \sqrt{183.36} = \cancel{14.27}\ 13.5$

$H_0 : \mu = 100$

$H_1 : \mu \neq 100$

$\alpha = 5\%$ with $df = n-1 = 10-1 = 9$

Table value $= 2.262$

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}} = \frac{(97.2-100)}{\frac{\cancel{14.27}\ 13.5}{\sqrt{9}}} = \frac{-2.8}{4.5} = -0.62$$

$|t| = 0.62$

Conclusion: cal t < Tab t.

We accept H_0.

Scanned by CamScanner

4. A certain injection administrated to each of 12 patients resulted in the following increase of blood pressure

5, 2, 8, −1, 3, 0, 6, −2, 1, 5, 0, 4

Can it be included that the injection will be, in general, accompanied by an increase in B.P ?

Soln:

Sample mean $\bar{x} = \dfrac{5+2+8-1+3+0+6-2+1+5+0+4}{12}$

$\bar{x} = 2.58$

Here $n = 12$

x	$x-\bar{x}$	$(x-\bar{x})^2$
5	2.42	5.8564
2	−0.58	0.3364
8	5.42	29.3764
−1	−3.58	12.8164
3	0.42	0.1764
0	−2.58	6.6564
6	3.42	11.6964
−2	−4.58	20.9764
1	−1.58	2.4964
5	2.42	5.85
0	−2.58	6.6564
4	1.42	2.0164

$\Sigma(x-\bar{x})^2 = 104.51$

$s^2 = \dfrac{\Sigma(x-\bar{x})^2}{n} = \dfrac{104.51}{12} = 8.71$ $\quad s = \sqrt{8.71} = 2.95$

1. $H_0 : \mu = 0$
2. $H_1 : \mu > 0$
3. $\alpha = 5\%$ with $n-1 = 12-1 = 11$ d.f

4. Table value 1.796

5. $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} = \dfrac{2.58 - 0}{\frac{2.95}{\sqrt{11}}} = 2.9$

6. Conclusion cal t ≮ table t.

Reject Ho. ie, we accept H_1.

Hence we may conclude that the injection is accompanied by an increase in B.P.

5. Sandal powder is packed into packets by a machine. A random sample of 12 packets is drawn and their weights are found to be (in kg) 0.49, 0.48, 0.47, 0.48, 0.49, 0.50, 0.51, 0.49, 0.48, 0.50, 0.51 and 0.48. Test if the average weight of the packing can be taken as 0.5 kg.

Soln: $n = 12$

$$\bar{x} = \frac{0.49+0.48+0.47+0.48+0.49+0.50+0.51+0.49+0.48+0.50+0.51+0.48}{12}$$

$$\bar{x} = \frac{5.88}{12} = 0.49$$

pop mean $\mu = 0.5$

x	$x-\bar{x}$	$(x-\bar{x})^2$
0.49	0	0
0.48	−0.01	0.0001
0.47	−0.02	0.0004
0.48	−0.01	0.0001
0.49	0	0
0.50	0.01	0.0001
0.51	0.02	0.0004
0.49	0	0
0.48	−0.01	0.0001
0.50	0.01	0.0001
0.51	0.02	0.0004
0.48	−0.01	0.0001
		$0.0018 = \Sigma(x-\bar{x})^2$

$$s^2 = \frac{\Sigma(x-\bar{x})^2}{n} = \frac{0.0018}{12} = 0.00015$$

$$s = 0.0122$$

$H_0 : \mu = 0.5$

$H_1 : \mu \neq 0.5$

$\alpha = 5\%$

$Df = n-1 = 12-1 = 11$

Table value $= 2.201$

Test Statistic $t = \dfrac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}} = \dfrac{0.49-0.5}{\frac{0.0122}{\sqrt{11}}} = \dfrac{-0.033}{0.0122}$

$t = -2.718$

$|t| = 2.718$

Conclusion : Cal t $\neq$ Table t

We reject Ho.